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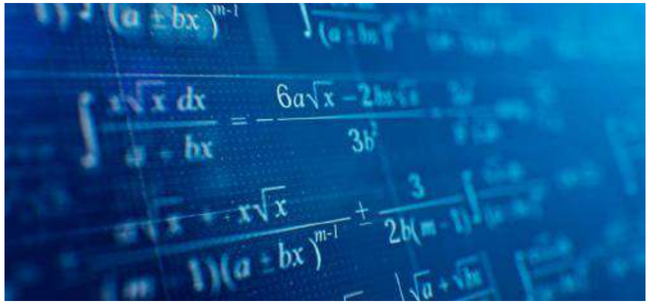

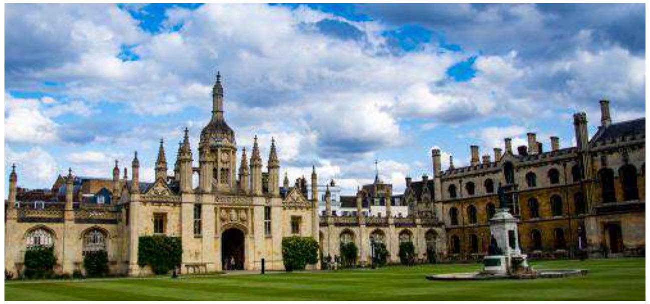

Mathematics

SYLLABUS OVERVIEW
16-18 YEARS OLDS

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About Immerse

Immerse Education is an award-winning academic summer school provider offering programmes for 16-18 year olds in centres of academic prestige.

The aim of these programmes is to provide participants with academically challenging content that develops their understanding of and passion for their chosen discipline. Through 40 hours of academic sessions, the programmes also offer young students unique and valuable insights into what it would be like to study their chosen subject at university.

This Syllabus Overview provides a summary of the topics and subject areas that participants can encounter during their studies with Immerse. It has been carefully created by our expert tutors who are current members of world-leading universities, and who have experience in teaching undergraduate students.

Academic Sessions

The academic sessions at Immerse are arranged into modules to enable participants to explore a broad range of topics over the course of two weeks. The modules included in this syllabus overview are indicative but not prescriptive.

Tutors are encouraged to include their own specialisms and also focus on any particular areas of interest expressed by participants within the class. They may choose to provide further detail on a specific topic, or they may include new material and information that builds on the knowledge already developed during the programme.

Personal Project

Each programme includes an element of individual work, generally termed the 'Personal Project'. This can take many forms but is commonly an essay or presentation delivered on the final day of the programme. Participants will receive feedback on this work which may also be mentioned in the participant evaluation which is provided in writing by the tutor once the programmes have ended.

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Preparatory work

Some tutors may ask participants to complete some preparatory work, such as reading or a series of exercises in advance of the programme. Participants are strongly encouraged to complete this work since it will be included in the opening sessions of the programme. Any preparatory tasks will be provided in advance of the programme directly to the participant.

Academic Difficulty

As all of our programmes are designed to provide a unique introduction to advanced material, the syllabus will be academically challenging at times.

This is something to be excited about and all of our tutors will encourage and support participants throughout the programme. Immerse Education aims to develop every participant regardless of ability, and our tutors will adapt their teaching to individual needs.

Aim of the Mathematics Programme

The Immerse Education Mathematics programme is designed to build upon the foundation of knowledge that participants have already gained in a traditional classroom environment and highlight how this can be used to inspire further study at university. Participants are encouraged to explore new material in-depth and to form independent and considered opinions and ideas based on sound academic knowledge and research. By the end of the programme, participants will have a good understanding, not only of university-level content, but also the variety of degree programmes available in subjects related to mathematics. Beyond this, participants will also explore the career opportunities available to graduates in this field.

Set Theory

In this session, participants will cover set theory from a more advanced standpoint: although seemingly intuitive and trivial, this branch of mathematics features remarkable ideas, and it not only lies at the foundation of all the other mathematical concepts, but it offers by itself beautiful ideas and constructions. Participants will be introduced to the theory before exploring a series of case studies and challenges designed to further their understanding and appreciation of this core mathematical concept.

Probability

The famous “Monty Hall problem” will tell you that the theory of probability is not always as simple as you may think, what does it really mean to say that two events occur independently (successively flipping a coin for example)? What is your probability of success in a game of dice? There are many problems in probability, and already interesting problems can be formulated using combinatorics, which deals with counting configurations of objects: for example, how many anagrams does a given word admit?

Integration

In one dimension, the integral of a function gives the area under a curve, and in two dimensions, the volume under a surface. Integration (often seen as the counterpart of differentiation) is one of the main tools of calculus, since one can use integration in many variables, generalising the intuitive notion of area and volume to that of higher dimensional objects (hypercubes, for example). We will introduce some powerful tools in integration: in particular, integration by parts and integration by substitution. We will also look into the approximation of integrals: this allows to calculate integrals of functions which cannot be expressed by simple formulae, and to provide a machine-implementable means to evaluate integrals.

Matrix Calculus

Gaining working knowledge of matrix calculus is an advantage when undertaking any scientific discipline. The yoga of matrix multiplication is easily understood, and it admits a beautiful geometric interpretation. We will start from linear systems, developing the formalism of matrices and learning to transform them using the Gauss-Jordan method. This leads to a very interesting classification of matrices, according to whether or not they can be ‘solved’ using this method. The underlying geometry is fascinating, and will be explored further when studying vector spaces.

Real Analysis

We are all familiar with the concept of numbers, like integers, rationals, reals. But how are they actually defined? We cannot build our mathematical castle only relying on intuition alone. It is essential for any university mathematics student to be able to write mathematics in a rigorous way; that is, in a way that is both mathematically valid, and understandable to fellow mathematicians. Such rigour is at the core of mathematical analysis, and is necessary to formulate any mathematical proof. In this module, we will consider the mathematical definition of sequences, series, limits of functions, and representing functions as infinite series.

Number Theory

In most real-world applications, we are familiar with dealing with real numbers. But integers still hold many mysteries, and are an interesting subject. First of all, we will explore the key concepts of divisibility and 'greatest common divisor' (GCD). An important tool is Euler's algorithm, which allows to compute the GCD of two numbers, and can be used in a creative way to solve diophantine equations. But there is more to see with integers. For example, on a 24-hour clock, if it is 7pm and we wait 10 hours, we reach 5am. In some way, we are saying that 19+10=5. By the same reasoning, 6*4=0. This is strange, since the product of two non-zero numbers should not be zero. How do we navigate this new world? The answer is modular arithmetic, which deals precisely with these kinds of 'numbers', with their strange rules of addition and multiplication.

TOPICS LIST

Group Theory

What do numbers and matrices have in common? They can be summed or multiplied: this means that there is an 'operation', which takes input of two 'objects' of a given kind, and returns a third of the same kind. But numbers and matrices are not the only ones. For example, look at a famous puzzle, the Rubik's cube: if you execute a move, followed by another move, you have in fact 'composed' them to produce a third move. This phenomenon is incredibly general, so general that it deserves a name: 'group'. We will see the rigorous definition of such a thing, and explore various examples. We will learn to easily represent groups symbolically, and to do computations in a completely formal and intuitive way: this is a very powerful tool.

Vector Spaces

There is a basic notion of vector, that is an oriented segment: this concept is familiar to all who have studied physics and dynamics, learning how to represent forces acting on an object. But this is just an example of what we might call a 'vector', and indeed a much more general theory holds. All linear equations, or systems, can be interpreted in terms of straight lines, planes in space, or intersections of such things. But why stop at three-dimensional space? We can go to any dimension, and we will learn to deal with higher-dimensional spaces. We will learn to represent a vector space (of any dimension) and its subspaces, using linear systems, or a set of 'elementary' vectors: this latter approach is similar to choosing two 'x' and 'y' axis in a Cartesian plane, but holds in much greater generality and is incredibly flexible.

TOPICS LIST

Octave for Mathematics

Octave is an easy to use software that will swiftly tackle the most challenging mathematical problems, as long as you tell it what to do. An understanding of how one formulates a mathematical algorithm is fundamental to achieving this goal, but it is much less daunting than it may sound. An algorithm can be as simple as just 'doubling the input' (e.g., \$2\ maps to \$4, \$4\maps to \$8). The more complex you make an algorithm, the more care and attention is required; for example, you may want to design an algorithm that repeatedly multiplies a number by \$2 until the outputted number is larger than \$10,000. This is simple to implement, but comes with pitfalls, for instance, what happens if you start with a negative number? Would this process be capable of generating a number that is larger than \$10,000? We will cover the basic syntax required to define simple mathematical algorithms, as well as how to understand 'for' and 'while' loops.

Modern Geometry

In this session, we will discover two key concepts at the foundation of contemporary geometry. The first is the projective space, which is, in some way, the study of 'geometry at infinity'. The second concept is that of the topological surface, that is a surface that can be deformed as if it were made of rubber: in this context, for example, a sphere and an empty cube are the same thing. This is the key concept of a branch of geometry called topology. We will see several examples of topological surfaces, how to tell them apart and how to easily construct and combine them. There are interesting features to be discovered.

Personal Project

Throughout the fortnight, participants will be working on their own personal project. Having been provided with a brief, participants should research and prepare a presentation for their peers. This will build upon an aspect of the theory that they have learnt over the course of the programme and is also an opportunity to showcase the academic research skills they have developed. The presentation is followed by questions from the audience and wider class discussion of particular points of interest. The tutor may also include feedback about the presentation in the written evaluation which is sent to participants after the programme has ended.

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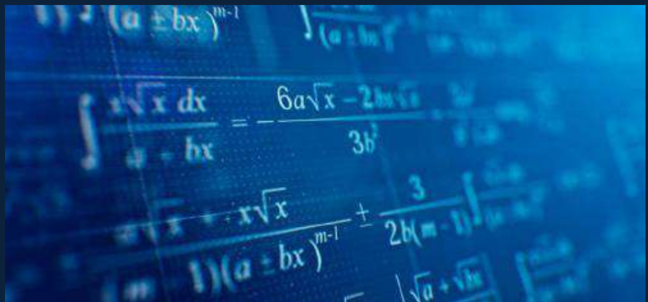

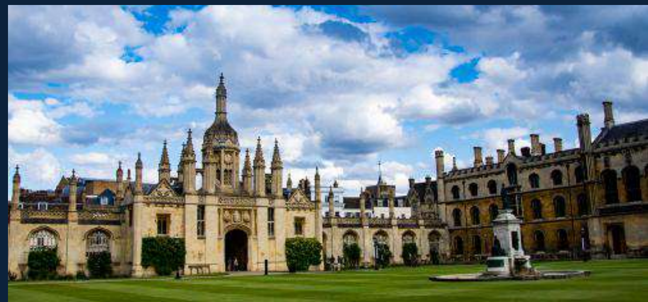


OUR AWARDS AND ACCREDITATIONS

THE QUEEN'S AWARDS FOR ENTERPRISE 2018

BAC ACCREDITED

COBIS SUPPORTING MEMBER COUNCIL OF BRITISH INTERNATIONAL SCHOOLS

ROSPA Member

BETA British Educational Travel Association